Consistent coupling of positions and rotations for embedding 1D Cosserat beams into 3D solid volumes

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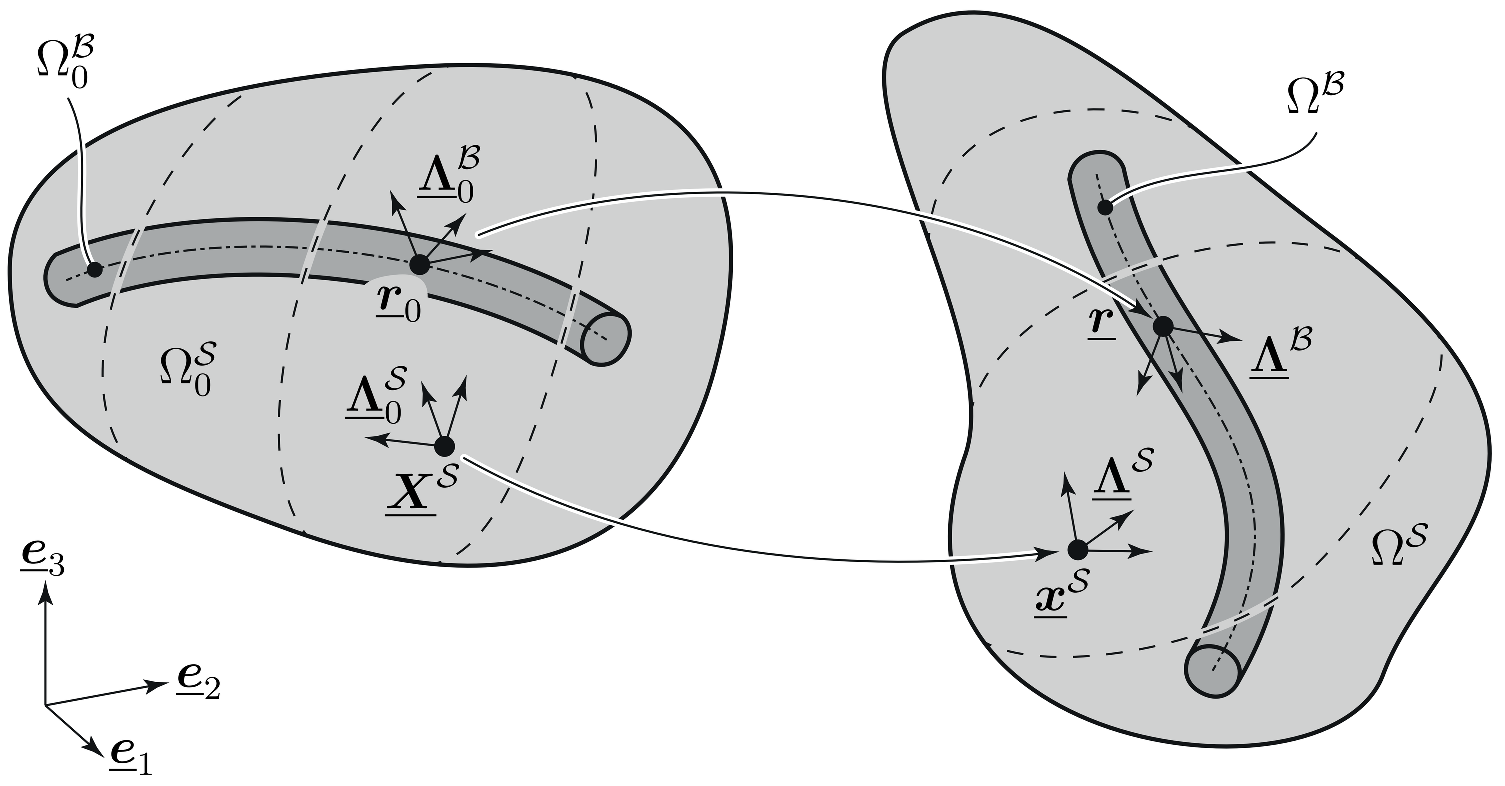
Motivation

Embedding 1D Cosserat continua (fibers / beams) [1] into 3D solid volumes requires a consistent method to model the mixed-dimensional coupling interactions between the two domains. Mechanically, the interaction between the fibers and the solid volume is defined on the 2D surface of the fibers, i.e., a 2D-3D coupling.



We approximate the 2D-3D coupling by a 1D-3D coupling along the centerline curve of the fiber Γ_c via,

$$\underline{\boldsymbol{r}} - \underline{\boldsymbol{x}}^{\mathcal{S}} = \underline{\boldsymbol{0}} \quad \text{on} \quad \Gamma_c$$
$$\underline{\boldsymbol{\psi}}_{\mathcal{SB}} = \underline{\boldsymbol{0}} \quad \text{on} \quad \Gamma_c.$$

The first set of constraint equations describes the coupling of the positions along the fiber centerline [2]. Only coupling the positions can lead to an underestimated stiffness of the compound structure, therefore, the second set of constraint equations enforces a coupling of the fiber cross-section rotations and the solid volume. This rotational coupling will be investigated here.

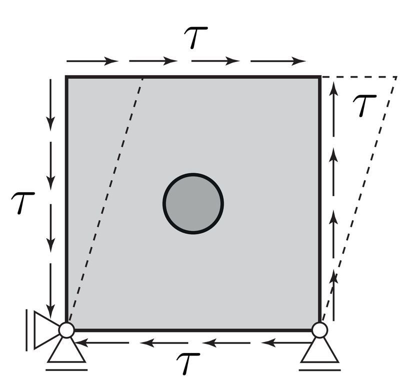
Solid Triad Field

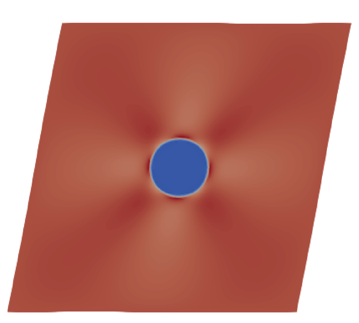
The rotational coupling constraints enforce a vanishing relative (pseudo)-rotation vector between a beam cross-section triad $\underline{\boldsymbol{\Lambda}}^{\mathcal{B}}$ and a corresponding soldid triad $\underline{\boldsymbol{\Lambda}}^{\mathcal{S}}$, cf. [3],

$$\underline{\boldsymbol{\psi}}_{\mathcal{SB}} = \operatorname{rv}\left(\underline{\boldsymbol{\Lambda}}^{\mathcal{S}} \underline{\boldsymbol{\Lambda}}^{\mathcal{B}\mathrm{T}}\right) = \underline{\boldsymbol{0}}.$$

One main contribution of this work is the definition of a suitable solid triad field in the solid (Boltzmann) continuum.

A **pure shear** problem is considered as a benchmark example (with a fine reference solution)



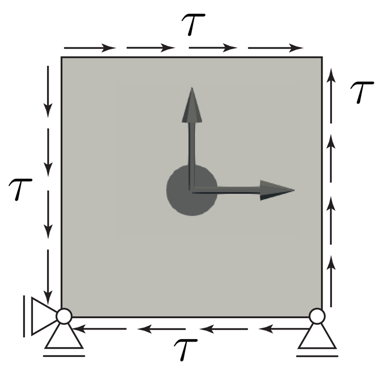


orthogonal triad

A first order Taylor series expansion of the 2D-3D coupling leads to the following solid triad fied:

$$\underline{\boldsymbol{\Lambda}}^{\mathcal{S}} = \underline{\boldsymbol{F}}$$

with the the solid deformation gradient $\underline{\boldsymbol{F}}$

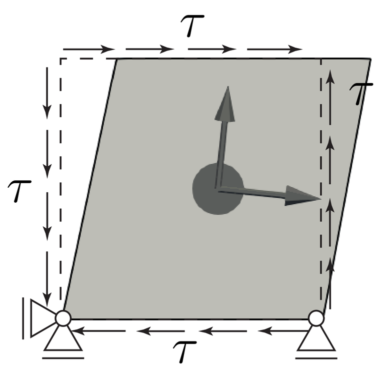


Catastrophic shear locking!

polar triad

As an alternative, the solid triad can be obtained as the rotational part of the polar decomposition of the solid deformation gradient $\underline{\boldsymbol{F}} = \underline{\boldsymbol{R}}\,\underline{\boldsymbol{U}}$:

$$\underline{\boldsymbol{\Lambda}}^{\mathcal{S}} = \underline{\boldsymbol{R}}$$



Very good agreement with the reference solution!

Discretization

- The rotational coupling constraints are enforced via the Lagrange multiplier method

$$\delta\Pi_\lambda^{\mathcal{R}} = \int_{\Gamma_c} \delta\underline{\boldsymbol{\lambda}}^{\mathcal{R}\mathrm{T}} \underline{\boldsymbol{\psi}}_{\mathcal{SB}}\,\mathrm{d}s + \int_{\Gamma_c} \underline{\boldsymbol{\lambda}}^{\mathcal{R}\mathrm{T}} \delta_o \underline{\boldsymbol{\psi}}_{\mathcal{SB}}\,\mathrm{d}s$$

- A mortar-type approach is used for the spatial discretization of the Lagrange multiplier field

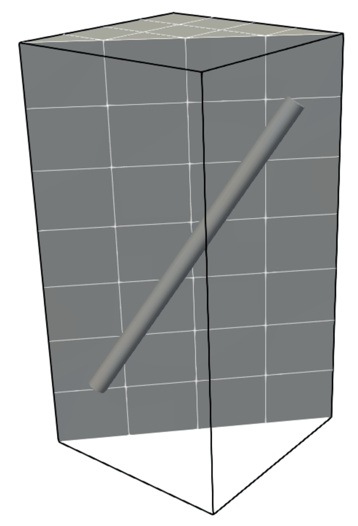
$$\underline{\boldsymbol{\lambda}}_h^{\mathcal{R}(f)} = \sum_{j=1}^{n^{\mathcal{R}(f)}} \Phi_j^{\mathcal{R}(f)}(\xi^{\mathcal{B}})\hat{\underline{\boldsymbol{\lambda}}}_j^{\mathcal{R}(f)}$$

- Global system of equations resulting from mortar-type discretization of rotational coupling and positional coupling (according to [2])

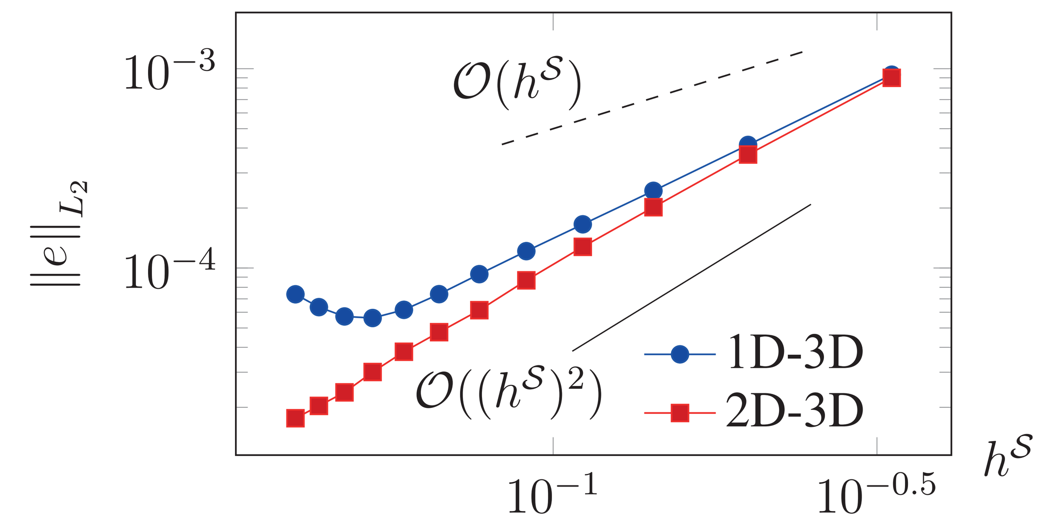
$$\begin{bmatrix} \mathbf{K}_{ss}^{\mathcal{S}} + \mathbf{Q}_{ss}^{\mathcal{R}} & \mathbf{0} & \mathbf{Q}_{s\theta}^{\mathcal{R}} & -\mathbf{M}^{\mathcal{V}\mathrm{T}} & \mathbf{Q}_{s\lambda}^{\mathcal{R}} \\ \mathbf{0} & \mathbf{K}_{rr}^{\mathcal{B}} & \mathbf{K}_{r\theta}^{\mathcal{B}} & \mathbf{D}^{\mathcal{V}\mathrm{T}} & \mathbf{0} \\ \mathbf{Q}_{\theta s}^{\mathcal{R}} & \mathbf{K}_{\theta r}^{\mathcal{B}} & \mathbf{K}_{\theta\theta}^{\mathcal{B}} + \mathbf{Q}_{\theta\theta}^{\mathcal{R}} & \mathbf{0} & \mathbf{Q}_{\theta\lambda}^{\mathcal{R}} \\ -\mathbf{M}^{\mathcal{V}} & \mathbf{D}^{\mathcal{V}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{Q}_{\lambda s}^{\mathcal{R}} & \mathbf{0} & \mathbf{Q}_{\lambda\theta}^{\mathcal{R}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{d}^{\mathcal{S}} \\ \Delta\mathbf{d}^{\mathcal{B}} \\ \Delta\boldsymbol{\theta}^{\mathcal{B}} \\ \boldsymbol{\lambda}^{\mathcal{V}} \\ \boldsymbol{\lambda}^{\mathcal{R}} \end{bmatrix} = \begin{bmatrix} -\mathbf{r}^{\mathcal{S}} \\ -\mathbf{r}_r^{\mathcal{B}} \\ -\mathbf{r}_\theta^{\mathcal{B}} \\ -\mathbf{g}^{\mathcal{V}} \\ -\mathbf{g}^{\mathcal{R}} \end{bmatrix}$$

Numerical Examples

consistency / verification

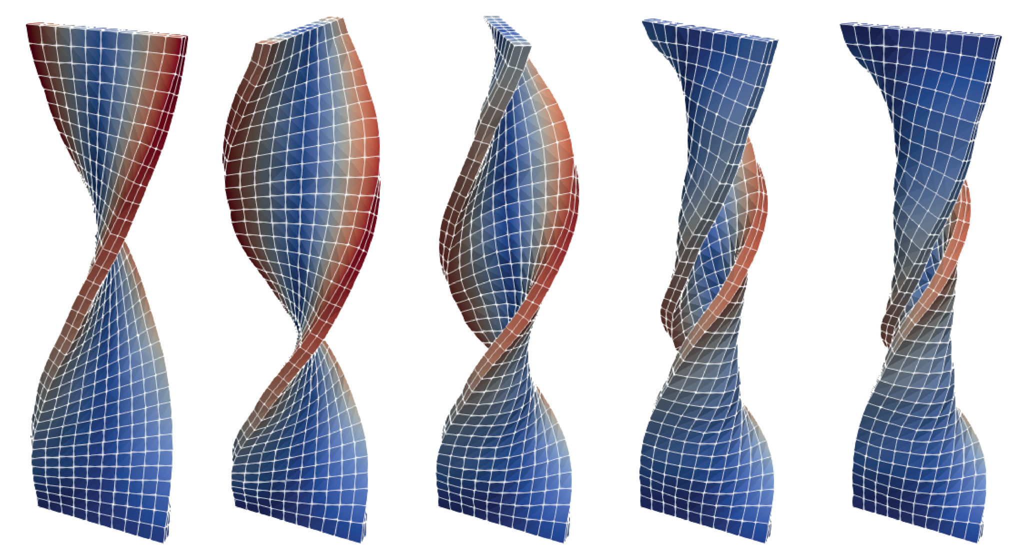


Patch tests are fulfilled

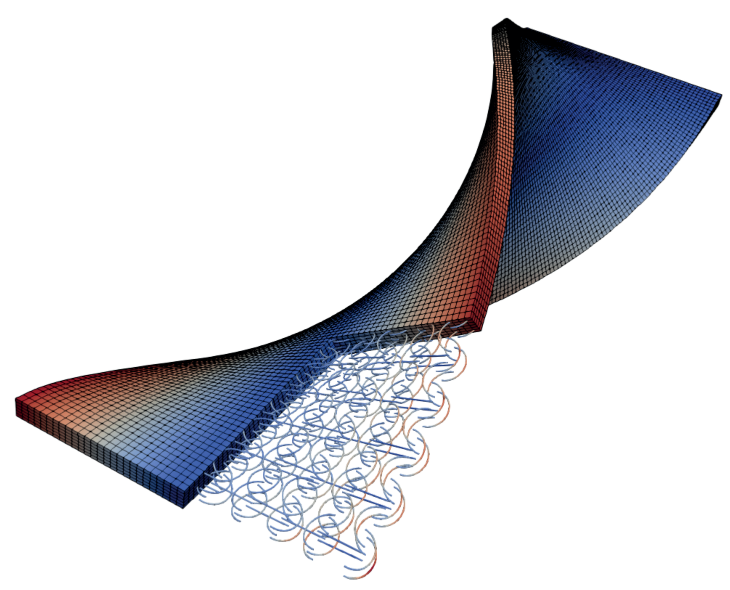


Spatial convergence for reasonable beam to solid element size ratios

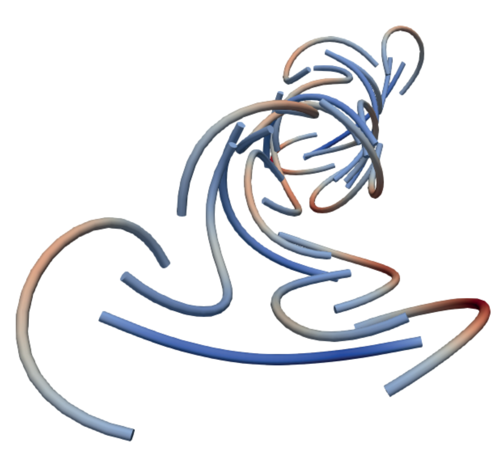
applications / validation

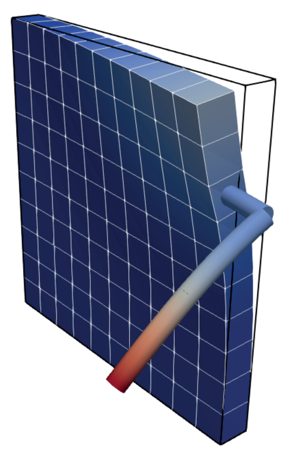


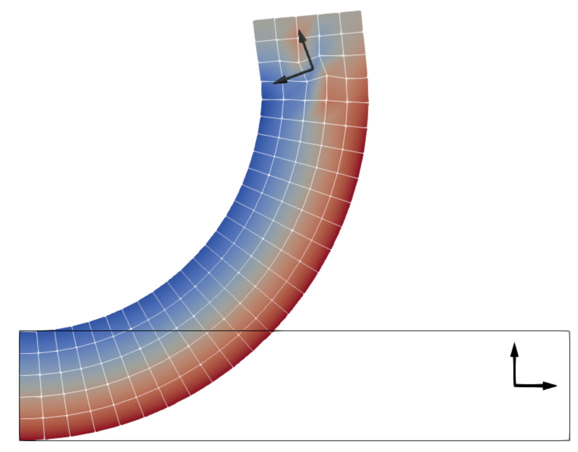
Twisted composite plate
Only positional *and* rotational coupling results in a correct stiffness of the compound strucutre



Large scale composite plate
The presented method is suitable for large scale appliacations







References

[1] Meier, C., Popp, A., Wall, W.A.: Geometrically exact finite element formulations for slender beams: Kirchhoff–Love theory versus Simo–Reissner theory. *Archives of Computational Methods in Engineering* **26**(1), 163–243 (2019)

[2] Steinbrecher, I., Mayr, M., Grill, M.J., Kremheller, J., Meier, C., Popp, A.: A mortar-type finite element approach for embedding 1D beams into 3D solid volumes. *Computational Mechanics* **66**(6), 1377–1398 (2020)

[3] Meier, C., Grill, M.J., Wall, W.A.: Generalized section-section interaction potentials in the geometrically exact beam theory (2021). Preprint, `https://arxiv.org/abs/2105.10032`